Angular Distribution of Muons in π-μ Decay at Rest

H. HULUBEI, J. S. AUSLÄNDER,* E. M. FRIEDLÄNDER, AND S. TITEICA†
Institute of Atomic Physics, Bucharest, Rumania

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Evidence for a significant departure from isotropy of the muon angular distribution from π-μ decay at rest is presented, based on measurements of projected angles in two emulsion stacks, exposed to the Dubna and CERN synchrocyclotrons. Extensive control experiments concerning observational bias, distortion, and inhomogeneous detection efficiencies prove that the observed lack of isotropy cannot be reduced to such spurious effects.

1. INTRODUCTION

AGAINST the general belief that the angular distribution of muons resulting from π-μ decay at rest must be isotropic, evidence has been brought forward repeatedly[1–6] for a marked departure from isotropy. These experiments, as well as most of the other, later emulsion and bubble chamber measurements,[7–22] irrespective of the conclusions arrived at by their authors, are open to various, serious objections, concerning statistics and/or observational and instrumental spurious effects. The only emulsion experiment[7] clearly free from such objections has yielded a significant lack of isotropy for low-energy pions from τ decay (see also reference 6). In the same paper[7] a negative result is reported from a counter experiment. These conflicting results suggest that experimental conditions (production, fields, etc.) might play an essential role.

In view of the quite peculiar conditions of exposure of the stack used in our previous experiments,[4–6] we deemed it necessary to perform a new set of measurements, designed so as to allow estimation of the possible contribution of spurious effects to the observed anisotropy or to avoid such effects. Furthermore, two plates of another stack, exposed at CERN under very similar conditions, have yielded an independent sample of π-μ decays, the angular distribution of which has been found significantly nonisotropic, too, and consistent with the results obtained from the main stack.

Anticipating our final conclusions, we must state that: (a) observational and instrumental effects, even if present, are unable to account for the observed lack of isotropy, and (b) the pion beams used in our experiments must have contained a fraction of pion-like bosons, with nonvanishing spin.

2. EXPOSURES

2.1. Dubna Exposure

The external 680-MeV proton beam of the J.I.N.R. synchrocyclotron, Dubna, has been directed on a $(CH_2)_n$ target, located outside the cyclotron fringing field (Fig. 1). The positive pions produced in the target were deflected by $\sim 22°$ in a deflecting magnet and led through a 2-m-long collimator into the experimental area, where they were detected by means of a scintillation counter telescope.

The magnet was tuned to 307 ± 10 MeV, which

* Present address: Polytechnical Institute, Bucharest, Rumania.
† Present address: Joint Institute for Nuclear Research, Dubna, U. S. S. R.

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[22] B. Bhowmik, D. Evans, and D. J. Prowse, see reference 21, p. IV–36.

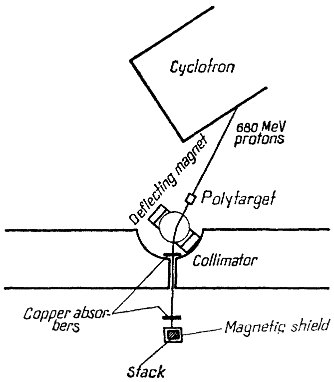

FIG. 1. Exposure conditions at the Dubna synchrocyclotron.

corresponds to the monoenergetic line of the reaction

$$p+p \rightarrow \pi^{+}+d; \quad (1)$$

the continuous background due to the reaction

$$p+p \rightarrow \pi^{+}+p+n \quad (2)$$

can be estimated roughly to $\sim 15\%$.

With the telescope, the range distribution of the pion beam in copper was measured behind the collimator. Consequently a copper absorber of 17 cm thickness was chosen as a moderator, of which 9.5 cm were placed before and 7.5 cm behind the collimator.

The pions were further slowed down by the front wall of a cubical iron shield, 2 cm thick ($H_{\max} < 0.2$ G[23]), which replaced the scintillators and contained a stack of nine NIKFI–R pellicles $10 \times 10 \times 0.04$ cm^3. The emulsion sheets had a horizontal position, their edges being parallel to the collimator axis which was contained in the plane of the central sheet.

The dip of the beam, measured near the edge of the stack, as expected, has been found to be negligibly small.

The energy spread of the beam at the stack was such that the distribution of pion stoppings was nearly uniform throughout the whole volume of the stack. The average density of pion endings was $\sim 10^4$ cm^{-3}. The sheets were mounted on glass backing after processing.

2.2. CERN Exposure

A similar exposure was made at the CERN synchrocyclotron (Fig. 2). The only differences were: forward pions from reaction (1), deflected 25° (energy 250 MeV), moderated by 12.5-cm copper plus 3-cm psendite plus equivalent 0.4-cm copper in scintillators, located in front of a double magnetic shield such that $H_{\max} < 0.3$ G.

The shield contained two stacks, a horizontal and a vertical one, of Ilford K5 pellicles, $7 \times 10 \times 0.06$ cm^3 each. The density of pion endings was ~ 80 cm^{-2}. The emulsions were mounted before processing.

[23] V. M. Sidorov (private communication).

One plate of the horizontal stack and one of the vertical stack were kindly lent to us by C. M. G. Lattes, who performed the exposure.

There are good reasons to assume that in the two experiments, conditions of production and exposure were quite similar. We believe that the essential and distinctive common features are: production *outside* the cyclotron field in a *two-particle reaction*, with *forward* emission.

3. ANGLE MEASUREMENTS

Scanning conditions and acceptance criteria will be described separately for each experiment.

All our angle measurements are concerned with projected angles in the emulsion plane which are referred to the y axis of the photographed coordinate grating which, incidentally, coincided within $\sim 2°$ with the axis of the pion beam at entrance into the stack. Obviously, in an infinite emulsion an isotropic angular distribution must yield a uniform distribution of projected angles, whatever reference direction is chosen.

A salient advantage of work with such projected angles is the absence of geometrical bias. Indeed, at first sight, it might seem that inclination of the beam with respect to the emulsion plane and nonuniform depth distribution of π-μ vertices might introduce a geometrical bias, viz., loss of forward events, which obviously cannot be corrected for by the usual double-scan procedure[24] for estimation of the true number of events.

In fact, the angular distribution of pion tracks at the end of the range is absolutely irrelevant for the question discussed here. We even may imagine for the sake of illustration that all pion tracks were eradicated, leaving arbitrarily distributed "pure vertices." Consider as an extreme case such vertices located exactly on either of the emulsion surfaces. In the hypothesis to be disproved (i.e., isotropy of muon emission) forward and backward events, as defined here with respect to a reference direction contained in the emulsion plane, will be equally probable and equally detectable.

All angle measurements have been performed by

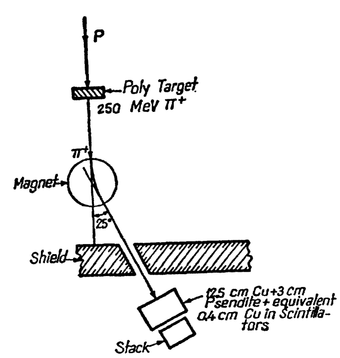

FIG. 2. Exposure conditions at the CERN synchrocyclotron.

[24] H. Geiger and K. Werner, Z. Physik 21, 187 (1924).

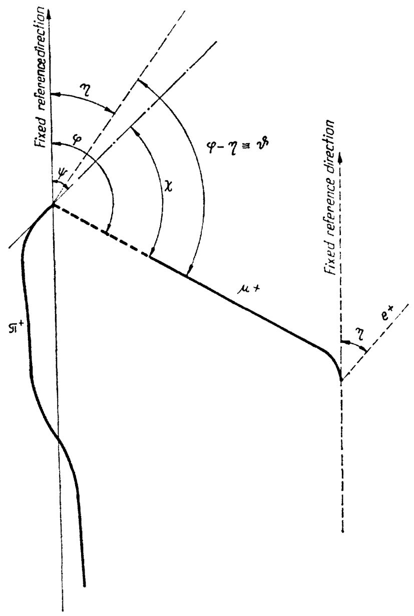

FIG. 3. Definition of measured angles.

means of eyepiece goniometers with an over-all accuracy of ~2°. The only exception is the first scan of our previous experiment, where "octants" of 45° were used.[4]

All pertinent angles are defined in Fig. 3.

4. SUMMARY OF OUR PREVIOUS EXPERIMENT (EXPERIMENT L)

Our previous results, described in detail (except for a slight increase in statistics) in references 4 and 5 (denoted further on as experiment L, i.e., low efficiency), are repeated here, in order to get a better synoptical view of the situation.

Since area scans might be subject to observational bias, two scans have been performed on the same area. The total number of events, R_i ($i=1$ or 2), found in scan No. i for a given angular interval has been split up into "single" events S_i, found only in scan No. i and "double" events D, found in both scans;

$$R_i=S_i+D,\quad (i=1 \text{ or } 2). \tag{3}$$

Then the usual procedure[24] leads to the following estimate Ω^* for the true number Ω of events to be expected in the given experimental conditions:

$$\Omega^*=R_1R_2/D=N+S_1S_2/D, \tag{4}$$

where

$$N\equiv S_1+S_2+D \tag{5}$$

is the total number of independent events accumulated in the double scan.

For the statistical error of this estimate see Appendix I. The numerical data are given in Table I, for 45°

TABLE I. Results of experiments L, T, α, H, and K-5 (angular distributions normalized to 1000).

Exp.		Over-all efficiency[a]	Sample size	I 0°–45° 315°–360°	II 45°–90° 270°–315°	III 90°–135° 225°–270°	IV 135°–180° 180°–225°	$b\times10^3$	$d\times10^3$	χ^2 (three degrees of freedom)	Optical[c] equipment
L	S_1	$\bar{P}_1^*=0.454$, $\bar{P}_2^*=0.665$	2900	193	259	273	274	-190 ± 37	-129 ± 37	51.4	Eyepieces 10×
	S_2		6937	217	260	263	260	-91 ± 24	-93 ± 24	40.5	Obj. 20×
	D		5750	189	290	291	229	-80 ± 26	-324 ± 26	171.2	
	Ω^*		19126[b]	205 ± 5	264 ± 5	269 ± 5	262 ± 7	-124 ± 21	-131 ± 21	106	
	$P_1^*\times10^3$			420 ± 10	480 ± 8	468 ± 8	423 ± 9				
	$P_2^*\times10^3$			661 ± 13	690 ± 11	679 ± 11	623 ± 12				
T			2875	242	236	278	245	-92 ± 34	-52 ± 37	12.3	Eyepieces 10×; obj. 20×
α			7357	255	255	242	248	$+40\pm23$	$+12\pm23$	3.5	Eyepieces 10×; obj. 20×
H	S_1	$\bar{P}_1^*=0.936$, $\bar{P}_2^*=0.979$	57	298	264	210	228	$+248\pm264$	$+104\pm264$	1.0	Eyepieces 10×
	S_2		180	272	261	183	284	$+132\pm149$	$+224\pm149$	4.5	Obj.: first scan 10×; second scan 60×
	D		2649	216	254	283	247	-116 ± 39	-148 ± 39	18.1	
	$N=\Omega^*$		2886	221	255	275	248	-95 ± 38	-124 ± 38	18.0	
	$P_1^*\times10^3$			920 ± 11	940 ± 9	959 ± 7	928 ± 10				
	$P_2^*\times10^3$			971 ± 7	978 ± 6	984 ± 5	980 ± 6				
K-5		$\bar{P}_1^*\approx0.69$ $\bar{P}_2^*\approx0.74$	8467	228	260	254	256	-48 ± 22	-64 ± 22	21.5	Eyepieces 10×; obj. 20×

[a] Weighted mean over quadrants I···IV.
[b] According to N. P. Klepikov [Zh. Eksperim. i Teoret. Fiz. 37, 1139 (1959)] 14 500 events would be necessary to ensure that the b value is significant on a 0.3% level.
[c] Intermediate magnification 1.5× throughout.

intervals of the angle φ, as well as the estimates Ω^* for Ω and P_i^* for the scanning efficiencies P_i:

$$P_{1,2}^* = D/(D+S_{2,1}). \qquad (6)$$

The departure from isotropy is characterized partially by two asymmetry coefficients, viz.,

$$b \equiv 2\times(\text{forward}-\text{backward})/(\text{forward}+\text{backward}), \qquad (7)$$

$$d \equiv 2\times(\text{equator}-\text{pole})/(\text{equator}+\text{pole}), \qquad (8)$$

("pole" means $90° \leqslant \varphi < 135°$), and globally by the χ^2 values of the Pearson test for uniformity of the angular distribution, with 3 degrees of freedom.

The figures of Table I show that even after the usual correction for an existing, but not essential bias, experiment L has yielded, from a purely statistical point of view, a significant departure from isotropy.

However, from a physical point of view two features of this experiment are liable to objections, viz., (a) discouragingly low over-all scanning efficiencies $P_{1,2}^*$; (b) the use of octants in the first of the two scans precludes estimation of Ω^* for small angular intervals; if the efficiencies P_i vary rapidly with φ, then Eq. (4), applied to 45° intervals, may yield misleading results.[25]

In order to meet these objections, other experiments with the same exposure and with another exposure were performed; their description and discussion is the object of this paper. Other possible objections, common to all area scan experiments, will be discussed in Sec. 8.

5. TRACK SCAN EXPERIMENTS

A first track scan experiment, denoted hereafter as experiment T_1, has been described briefly in reference 6 (NIKFI emulsions). In order to emphasize its freedom of observational bias, we give here the scanning procedure in more detail.

Grey parallel tracks were picked out of the pion beam in a given strip of the emulsion and followed in the beam direction from plate to plate until decay, stopping, or exit from the stack.

All decays occurring in the same plate in which the track had been picked up were rejected in order to prevent scanners from being influenced by the shape of the decay event. Each stopping track was carefully inspected under high magnification for π-μ decay (characteristic change of ionization and Coulomb scattering).

Each muon leaving the plate in which the π-μ apex had been found was followed into the neighboring ones to the point of μ-e decay. Whenever a track ended directly by decay into an electron, it was followed back some 600 μ and reinspected there for a possible π-μ apex. If no such apex was found, the event was classified as a beam muon. From all accepted grey beam tracks, $\sim$3% turned out to be beam muons, $\sim$8% protons, $\sim$12% were not found in the next sheet (heavy background of grey beam tracks), $\sim$3% had no detectable electron, $\sim$22% left the stack, and $\sim$52% proved fit for measurement. No muon tracks from π-μ decay at rest were lost.

Unfortunately, this experiment could not be continued in its initial form, since several plates of the stack deteriorated accidentally. Consequently, scanning instructions had to be changed. Tracks, which according to a visual, rough estimate of ionization and scattering appeared to be slow pions, were followed within one emulsion sheet until exit or stopping.

Only π-μ-e events contained completely in this emulsion sheet were accepted; in order to keep the scanners free from any influence of the shape of the π-μ decay, all events were discarded for which this decay occurred less than 1 mm from the point at which the track had been picked up.

In this experiment, T_2, the total number of events accepted according to the above criteria, was 1141; the fraction of decaying beam muons was $\sim$5%.

The results for T_1, already reported in reference 6, and those for T_2 are listed together under T in Table I.

Under such scanning conditions it is hardly conceivable that observational bias could have played any role. Nevertheless, to disprove the possibility that the observed anisotropy resulted from misclassification of π-μ-e decays with small angle χ as beam muons, the following test was performed.

Each event, whose classification as π-μ-e or as beam μ-e could be considered as doubtful, was inspected by 2 or 3 persons of which at least one was an experienced physicist. Furthermore, in experiment T_2, gap counts were performed on twelve adjacent cells of 100μ dip-corrected track length, starting from the μ-e apex, for (a) 18 tracks classified as beam muons, (b) 32 tracks classified as π-μ-e decays with small angle χ (see Fig. 3), and on (c) 28 obvious π-μ-e decays with $\varphi > 135°$. The results of these gap counts are plotted in Fig. 4 which clearly shows that practically no forward π-μ decays have been erroneously rejected as beam muons. Otherwise, curves (b) and (c) of Fig. 4 could not show such similar behavior, nor could curve (a) of Fig. 4 be so strikingly different from curves (b) and (c).

A Kolmogorov-Smirnov test[26] proves consistency of (b) and (c) on a 99.99% probability level and striking incompatibility between (a) and (b)+(c), viz., probability of consistency $\ll 10^{-8}$.

An upper limit to the fraction of π-μ decays wrongly classified as beam muons (and thus lost in the first 45° interval) can be set in the following way. Define a "muonity" parameter δ_a for curve (a) of Fig. 4, viz., the difference between the areas on the histogram located right and left of the abscissa 600 μ. Assume now that curve (a) (supposed to be beam muons) consists actually of a fraction x of π-μ decays which

[25] C. J. Waddington, Suppl. Nuovo Cimento **19**, 37 (1961).

[26] B. L. van der Waerden, *Mathematische Statistik* (Springer-Verlag, Berlin, 1957), Chap. XI, Sec. **56**.

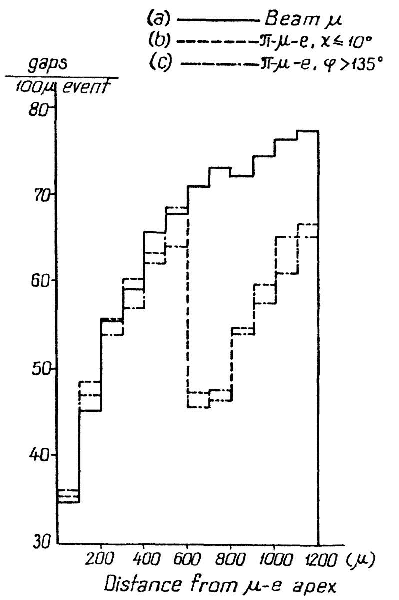

FIG. 4. Results of gap counts for discrimination between π-μ decays and beam muons.

were erroneously considered as beam muons and a fraction $(1-x)$ of true beam muons. Then δ_a can be considered as a linear combination of the quantity δ_c, defined similarly for the pure π-μ decay sample of curve (c), and an unknown quantity δ_μ, defined analogously for a pure muon sample. An upper limit for x is obtained by the assumption that δ_a and δ_μ are statistically indistinguishable. Using the data of Fig. 4, there results $x \lesssim 0.2$ on a 99.7% confidence level. As the fraction of muons in the pion beam is at most $\sim 5\%$, this value of x means that the fraction of π-μ decays lost in this way is surely less than $\sim 1\%$.

In spite of the very stringent acceptance criteria and of the relatively low statistics entailed by these precautions, a deviation from isotropy as large as the one observed in sample T is expected to occur as a statistical fluctuation with a probability of at most 0.7% (see χ^2 value in Table I). The b value for this sample is different from the value zero expected for isotropy by 2.48 standard deviations; the one-sided probability of a deviation as large as this or larger is again $<0.7\%$.

Thus, it seems extremely improbable that the angular distribution of muon *tracks* be isotropic. Since, in view of possible distortions of the emulsion, this conclusion need not be true necessarily for the object of physical interest, viz., the angular distribution of initial muon *momenta*, a set of control experiments has been performed (see Sec. 8 and especially Sec. 6).

The first of these consisted in recording in the same plates, with the same scanners and microscopes, the angular distribution of tracks due to particles known *a priori* to be distributed isotropically. For a meaningful comparison of these objects with the muons from π-μ decay, a sample of such decays is needed, which must satisfy the following conditions: (a) The π-μ decays must be located in the immediate vicinity of the control object; (b) they must be detected without observational bias; (c) the statistics must be large enough. As reference objects we chose α prongs from radioactive contamination stars. Since the probability to find an α star within, say, $\sim 500\,\mu$ from a π-μ apex of experiment T is exceedingly low, conditions (a) and (c) cannot be met by a track scan.

Consequently, a new area scan, designed so as to combine good statistics with high scanning efficiency, was performed (see the next section).

6. HIGH-EFFICIENCY AREA DOUBLE SCAN (EXPERIMENT *H*)

The NIKFI plates were area scanned for contamination stars with at least 3 prongs. The projected angles of all α prongs with respect to the positive y axis of the coordinate grating were recorded by means of octants.

Each 1-mm² cell of the grating containing at least one such star was scanned twice with special care and reduced scanning speed for π-μ decays.

In the first scan a thorough "map" of each cell was made, so that actually it was inspected several times. Then a second scan was performed under higher magnification (oil immersion). From the events recorded in this way, only complete π-μ-e decays in the same sheet were accepted for measurement.

The results are given in Table I, under the headings α and H. It can be seen that the angular distribution of α prongs is practically isotropic. If the b value is to be taken at face value, it is of opposite sign to all π-μ results. Isotropy of α prongs in other plates of the same stack has been reported also by other authors.[20] The same result has been outlined in another batch of NIKFI–R plates soaked with a solution of $Th(NO_3)_4$.[27] As to the muon sample of experiment H, it is significantly nonisotropic, the shape of its distribution being the same as that of experiments L and T.

As a consequence of the high efficiencies, the term S_1S_2/D of Eq. (4) is negligibly small (<2 events per 45° interval). Hence, practically for experiment H

$$\Omega^* = N. \qquad (9)$$

As a sum of Poisson distributed quantities [see Eq. (5)], N is itself Poisson distributed. The standard deviations pertaining to this experiment have been computed accordingly.

7. EXPERIMENT IN ILFORD K-5 PLATES

Taking advantage of the fact that an Ilford K-5 stack had been exposed by Lattes and processed by Vanderhaeghe at CERN (Sec. 2.2) under conditions very similar to ours, we recorded the projected angular distribution of π-μ decays in two of these plates, a horizontal and a vertical one, too.

[27] D. Anghelescu, J. S. Ausländer, I. I. Georgescu, and A. Vogel, Rev. Phys. Acad. Rep. Populaire Roumaine **6**, 259 (1961).

TABLE II. Results of restricted double scan in Experiment K-5 (angular distributions normalized to 1000).

	Sample size	I	II	III	IV	$b\times10^3$	$d\times10^3$	χ^2
S_1	121	27	36	33	25			
S_2	148	22	49	49	28			
D	341	62	100	84	95	-100 ± 108	-160 ± 108	
N	610	111	185	166	148	-60 ± 81	-310 ± 81	22.4
S_1S_2/D	54	10	18	19	7			
Ω^*	(664)	121 ± 12	203 ± 15	185 ± 15	155 ± 13	-48 ± 84	-338 ± 84	22.4
P_1^* (%)	($\bar{P}_1^*=69\%$)	74	67	63	77			
P_2^* (%)	($\bar{P}_2^*=74\%$)	70	73	72	79			

The plates were scanned by two scanners, who were shifted daily from one plate to the other.

Figure 5 shows a microprojection drawing of a typical π-μ event in these plates. It is obvious that the ionization gradients in the vicinity of the π-μ apex are so strong that any loss of forward decays (such a loss might be possible in Ilford G-5 or NIKFI–R emulsions) is practically precluded here. Under such conditions a single scan seems to be sufficient in order to establish whether the angular distribution in this exposure is similar to ours. In view of the low beam intensity and the good discrimination, we accepted complete and incomplete (escaping muons) π-μ decays without distinction, in order to enlarge the statistics.

The results are given in Table I under K-5. The data from the horizontal and the vertical plate are pooled, as they were found to be consistent within statistics.

In order to gain an idea about the efficiency, we scanned a restricted area for a second time under identical conditions. The results of this control scan are given in Table II.

As can be seen, the lack of events in the first quadrant cannot be ascribed to bias. This conclusion results also from the fact that the equator-pole asymmetry coefficient for the "double" events,

$$d_D=-0.160\pm0.108,$$

is, if anything, lower than the corresponding value for the whole sample of independent events [see Eq. (5)]:

$$d_N=-0.310\pm0.084,$$

which even at such low statistics is significantly different from zero.

For comparison see the reverse situation in the low-efficiency experiment (Table I).

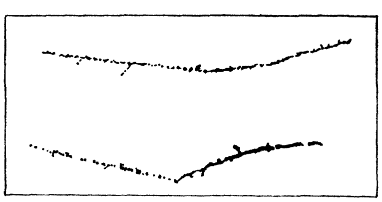

FIG. 5. Microprojection drawing of typical forward π-μ decays in (a) NIKFI–R and (b) Ilford K-5 emulsion.

8. CONTROL EXPERIMENTS

The control experiments described below were designed so as to obviate remaining doubts concerning: (i) wrong identification of π-μ apex, (ii) distortion of the emulsion, and (iii) failure of the double scan procedure to correct for observational bias, due to nonhomogeneous detection efficiencies.

8.1 Possible Confusion of Part of Muon Track with Pion Ending

Such a confusion might arise either by large-angle scattering at $\sim600\,\mu$ residual range of muons which normally contaminate the pion beam or by similar scattering of π-μ muons in the first grains of their track.[2,18]

In the course of the track scanning the muon contamination has been estimated to be ~3–5%. Furthermore, it has been shown[28] that the fraction of muon scatters by >5° at $\sim600\,\mu$ is negligibly small (see Fig. 6). For contamination muons this small effect would be of opposite sense to the observed one, while for π-μ muons it would average out.

It is interesting to note that in bubble chamber experiments contamination muons, more difficult to detect, may play a larger part. This seems to have been the case with two experiments[12,13] carried out with the same beam of the Dubna accelerator under very similar conditions, where large-angle scattering of contamination muons has been invoked for conflicting interpretations.

As to our data, the correct identification of π-μ decay points has already been proved independently by the gap counts described in Sec. 5.

8.2 Distortion

The α-prong experiment described in Sec. 6 is in itself a proof for the fact that the departure from isotropy, observed in the distribution of projected muon angles in our NIKFI stack, is not simulated by distortion. Further evidence in support of this conclusion comes from the following control experiments.

8.2(1) In experiment L (low-efficiency area scan), in which only complete π-μ-e events were accepted, the

[28] E. M. Friedländer, Rev. Phys. Acad. Rep. Populaire Roumaine 5, 355 (1960).

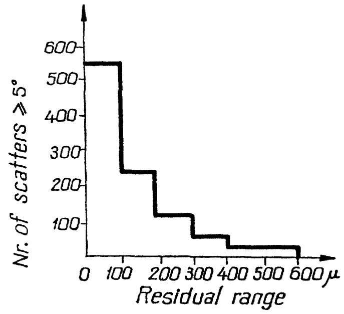

FIG. 6. Frequency of scatters $\geqslant 5°$ on muon tracks from 8118 π-μ decays at rest vs residual range at scattering point.

projected angles ϑ between the decay electron and the initial muon momentum were recorded by the same scanners immediately after recording the angles pertinent to π-μ decay.

The distribution of these angles is given in Fig. 7. From these data we computed,[11] taking into account the finite thickness of the emulsion, the parameter of the distribution of spatial angles (including the residual degree of polarization):

$$a = -0.069 \pm 0.019,$$

in good agreement with the result of Gurevich *et al.*,[29] obtained under very similar conditions in the same type of plates (NIKFI–R), in which (as asserted in reference 29) the depolarization is stronger than in Ilford emulsions.

Now, if part of our "pions" have nonvanishing spin, there is no reason to expect the same μ-e angular distribution as for zero-spin pions (except, perhaps, under very special conditions of polarization). In our previous paper[4] we have presented some preliminary evidence for the possibility that the $(1+a\cos\vartheta)$ distribution is a superposition of partial distributions, depending on the muon emission angle. Considerably larger samples are, however, needed in order to settle this problem. The same is true for the departure from the $(1+a\cos\vartheta)$ form of the global distribution, which departure might be small enough to escape detection with our present statistics.

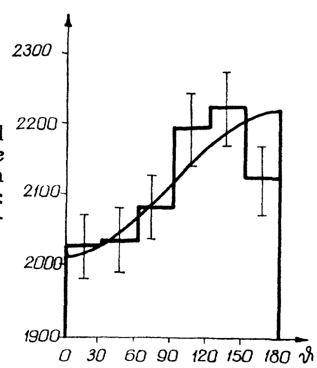

FIG. 7. Histogram: Observed angular distribution of μ-e decays (see Fig. 3 for definition of angle ϑ). Continuous line: best fit $(1+a\cos\vartheta)$ distribution.

[29] I. I. Gurevich, V. M. Kutukova, A. P. Mishakova, B. A. Nikolsky, and L. V. Surkova, Zh. Eksperim. i Teor. Fiz. 34, 280 (1958).

8.2(2) A stack used earlier in our laboratory[3] had incidentally been irradiated twice in the same pion beam, the difference between the two exposures being that the pions entered the stack from two opposite edges.

We recall that all muon angles considered hitherto were defined with respect to a direction rigidly connected with the plates, viz., the positive y axis of the coordinate grating. The same is true for the muon angles measured in the earlier stack mentioned above.

Assume now tentatively an isotropic muon momentum distribution and a corresponding track distribution, which due to distortion has become nonisotropic.

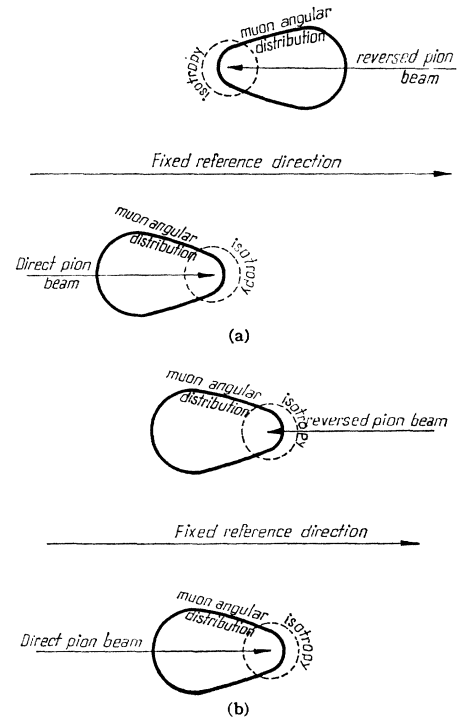

FIG. 8. Schematic illustration of muon angular distributions expected in plates irradiated by a direct *and* by a reversed pion beam, if the anisotropy is due (a) to distortion of the emulsion; (b) to a genuine property of the pions.

Obviously distortion affects the muon track, irrespective of the direction of motion of the parent pion. Hence, if due to distortion a backward excess with respect to the fixed reference direction has appeared for one of the pion "beams," the same backward excess (referred to the same reference direction) must appear in the opposite "beam," too. This situation is illustrated schematically by the polar diagrams of Fig. 8(a).

If, instead, the asymmetry is due to a property of the pion, it must be connected with the pion momentum and not with the arbitrary orientation of the plates. Hence, the sign of the asymmetry ought to be the same for both "beams" if it were referred to the pion momentum at entrance into the stack, but it must be of opposite signs in the two beams if referred—as in our case—to the fixed reference direction [see Fig. 8(b)].

Experimentally we found

$$\begin{aligned} &\text{for beam 1,} \quad b_1 = -0.143 \pm 0.038; \\ &\text{for beam 2,} \quad b_2 = +0.076 \pm 0.051. \end{aligned} \tag{10}$$

The departure from zero of the difference $b_2 - b_1 = 0.219 \pm 0.064$ is highly significant. This is just the situation expected if the asymmetry is due to a physical property of the pion [Fig. 8(b)].

Reversing the reference direction for beam 2 and adding then the two angular distributions, a global value b' of the asymmetry is obtained:

$$b' = -0.118 \pm 0.031, \tag{11}$$

which is in best agreement with our previous and present results.

This is obviously the case also for b_1 and b_2, taken separately.

8.2(3) It has been shown by Castagnoli *et al.*[19] that if distortion is responsible for the apparent forward-backward muon asymmetry, then this asymmetry must be of opposite sign for muons pointing towards the glass and towards the air interface.

A check for the presence or absence of this effect in our NIKFI stack has been performed with 10 889 events, from experiment L, for which it had been recorded whether the muon track pointed towards the air interface ("up"), towards the glass interface ("down"), or was practically horizontal ("zero"). The corresponding asymmetry coefficients are:

$$\begin{aligned} b_{\text{up}} &= -0.087 \pm 0.038, \\ b_{\text{down}} &= -0.094 \pm 0.027, \\ b_{\text{zero}} &= -0.141 \pm 0.040. \end{aligned} \tag{12}$$

The probability that these three values deviate from one another because of purely statistical fluctuations about a common mean (-0.103 ± 0.019) is measured by a Pearson test, yielding $\chi^2 = 1.19$ with two degrees of freedom. Thus, distortion as a cause of the observed asymmetry is again disproved.

8.3 Inhomogeneous Detection Efficiencies

The quantity Ω^* defined by Eq. (4) is an estimate for Ω, if—and only if—the detection efficiencies P_1 and P_2 are the same for *all* events of the sample under consideration. Otherwise, as can be easily shown, Ω^* is an estimate of the quantity

$$\Omega \frac{\bar{P}_1 \bar{P}_2}{\langle P_1 P_2 \rangle_{\text{av}}} \equiv \Omega Y, \tag{13}$$

the bars and angular brackets referring to averaging over the *functional* dependences of P_1 and P_2 on different physical parameters, distinguishing some of the events from other ones. Always $Y \leqslant 1$, i.e., Ω^* underestimates Ω.

It will be shown in a forthcoming paper[30] that a general criterion based on the statistical correlations between the "single" events S_1 and S_2 can be derived, which is—in principle—able to distinguish between homogeneous and inhomogeneous detection efficiencies. In principle, this method reveals existing unhomogeneities, irrespective of the number and kind of the physical parameters causing it. In practice, however, in order to lead to statistically significant conclusions it needs much larger statistics than are available in this experiment.

In this situation all one can do is to make a guess as to the nature of some of the parameters which may be reasonably suspected to cause inhomogeneities, and to investigate their influence.

8.3(1) As can be seen in Table I, the efficiencies P_1^* and P_2^*, estimated per quadrant, are almost independent of angle. It is conceivable, however, that this is due to a wrong estimation of the P_i^*, caused by a fast variation of the P_i in small angular intervals which is drowned in the estimation per quadrant.

To check this possibility we have split up the data of sample H into 5° intervals of φ and have computed the quantities S_1S_2/D, i.e., the correction term for bias in Eq. (5), for each of these intervals. If the above inhomogeneity were operating, then the quantities S_1S_2/D per 5°, added for whole quadrants, ought to exceed considerably the quantity S_1S_2/D per 45°; i.e., for the first quadrant:

$$\sum_{0^\circ}^{45^\circ} S_1S_2/D > \sum_{0^\circ}^{45^\circ} S_1 \sum_{0^\circ}^{45^\circ} S_2 / \sum_{0^\circ}^{45^\circ} D, \tag{14}$$

and similarly for other quadrants. Furthermore, if the observed lack of isotropy were due to such effects, the inequality (14) would be expected to be especially strong for the first quadrant.

In fact, Table III shows that: (i) the left-hand side and the right-hand side of Eq. (14) are practically equal; (ii) the first quadrant does not differ in this respect from the other ones; (iii) none of the values S_1S_2/D exceeds two events per quadrant, so that either way the correction is completely negligible.

As a supplementary precaution we studied the angular distribution of the first quadrant. Table IV

TABLE III. Comparison of over-all and cumulated correction terms in experiment H, in events per quadrant [see Eq. (14)].

φ	I	II	III	IV
$\Sigma S_1S_2/D$	1.55	0.68	0.28	0.93
$\Sigma S_1 \Sigma S_2/\Sigma D$	1.48	0.67	0.31	0.85

[30] J. S. Ausländer, E. M. Friedländer, and H. Totia (to be published).

TABLE IV. First quadrant of experiment H, number of events per 5° intervals.

φ^0	0–5	5–10	10–15	15–20	20–25	25–30	30–35	35–40	40–45	0–45
N	65	79	68	86	83	65	56	77	54	638

shows that, for example, in the 30°–35° interval there appear fewer events than in the 0°–5° interval. Now, assume nevertheless that the first 5° interval be underpopulated, due to loss of "straight" events. The mean population per 5° interval for the first quadrant, excluding the first 5° interval, is 72±3 events, as compared to 65±8 events actually found between 0° and 5°.

Increasing now the interval involved by three standard errors of the difference, one obtains as an upper limit for the true population of the first quadrant 663 events, instead of 638 found. Even after such an exaggerated correction the Pearson test against isotropy still yields $P_{\chi^2}\approx 3\times 10^{-3}$.

8.3(2) Obviously the greatest danger of loss concerns straight events, i.e., π-μ decays with small angles χ (see Fig. 3). Such angles occur mainly for $\varphi<45°$ in view of the strong forward collimation of the pion beam. Hence, especially the first quadrant might be affected by an inhomogeneity which would imply a residual loss uncorrected for by the double-scanning procedure. The influence of such an uncorrected loss on the forward-backward asymmetery can be estimated approximately as follows:

Let us make the simplifying assumption that all events with $|\chi|$ less than some angle χ_0 are lost (detection efficiency zero) while the rest of the events are detected with efficiency p^*. In order to estimate the fractional loss of events in any angular interval of φ, one has to fold the distributions of ψ and φ (Fig. 3), for a given χ, and to integrate from $\chi=-\chi_0$ to $\chi=\chi_0$.

The angular distribution of pion endings (ψ) has been shown in reference 28 to be of Gaussian shape, with an rms angle $\Delta\approx 30°$, which is independent of the location in the plate. This means that angles $\psi>90°$ are practically absent (>3 standard deviations), and hence all losses are concentrated in the forward quadrants.

It can be shown that the fractional loss Z for the interval $0\leqslant\varphi<90°$ is given by

$$Z=\frac{\Delta}{\pi}\left[F(0)+F\left(\frac{\pi}{2\Delta}\right)+\frac{\chi_0}{\Delta}\Phi\left(\frac{\pi}{2\Delta}\right)-F\left(\frac{\chi_0}{\Delta}\right)-F\left(\frac{\pi/2-\chi_0}{\Delta}\right)\right], \quad (15)$$

where

$$F(y)\equiv y\Phi(y)+[2/(2\pi)^{1/2}]e^{-y^2/2}, \quad (16)$$

and $\frac{1}{2}\Phi(y)$ is the error integral.

From previous data concerning angles ψ (reference 4), measured on 8118 events which constitute the major part of the L events described in the present paper, we deduce an efficiency $p^{**}=0.37\pm0.04$ for events with $|\chi|<10°$ and $p^*=0.434\pm0.006$ for the rest. From these values we can estimate χ_0:

$$\chi_0=10°\times(p^*-p^{**})/p^{**}=(1.6\pm1.1)°. \quad (17)$$

Thus, from a statistical point of view, the totally ineffective angular interval χ_0 is well consistent with zero. Nevertheless, we take Eq. (17) at face value and apply the correction of Eq. (15). The value of Δ has been checked for the 8118 events used here and found equal to $(29.9\pm0.3)°\approx 30°$.

Using these numerical values one obtains the apparent forward-backward asymmetry caused by $\chi_0\neq 0$:

$$b_{\text{app}}\simeq(-9\pm16)\times 10^{-3}, \quad (18)$$

whence, the remaining forward-backward asymmetry of sample L is

$$b_{\text{corr}}=(-113\pm24)\times 10^{-3}, \quad (19)$$

which is still highly significant.

It is perhaps not without interest to mention that actually 81 events have been recorded with $|\chi|<1.5°$, while 56 are to be expected if constant efficiency is assumed throughout.

8.3(3) A further possible inhomogeneity can be due to different efficiencies for different scanners. For each of the six scanners in experiment H, we have computed individual values of P_1^* and P_2^* per quadrant. The resulting Y values [Eq. (13)] are given in Table V.

8.3(4) Reduced contrast near the bottom of the plate may also introduce an inhomogeneity in detection efficiencies. We have not measured explicitly the depth distribution of π-μ vertices but the data given under 8.2(4) ("up," "down," and "zero" events) may be used as an indirect check against this effect. Indeed, in view of the finite emulsion thickness (400 μ) and the requirement of complete π-μ-e chain within one emulsion sheet, it is obvious that π-μ decays with vertex near the bottom are predominant among "up" events, etc.

If the observed asymmetry were due to this effect, one would expect

$$|b_{\text{up}}|>|b_{\text{zero}}|>|b_{\text{down}}|. \quad (20)$$

TABLE V. Test for homogeneity with respect to scanners: the quantity Y [Eq. (13)] estimated per quadrant for experiment H.

Quadrant	I	II	III	IV
Y	1.0001	1.0005	1.0016	1.0007

The Pearson test given under 8.2(4) disproves any systematic variation of b and the numerical data of Eq. (12) show, if anything, a situation reversed with respect to inequality (20).

8.3(5) Finally, another possible cause of inhomogeneity has been investigated, viz., busyness of the field of view. Scanners might be inclined to lose forward events preferentially in busy fields (heavy background of grey tracks).

As has been mentioned in Sec. 6, in experiment H, all π-μ events were recorded initially, irrespective of the fate of the muon (exit from the sheet or μ-e decay within).

The number m of such (complete and incomplete) π-μ events per 1-mm^2 cell has been considered as a convenient measure for busyness. The cells were classified as follows: (1) $m\leqslant 5$, (2) $6\leqslant m\leqslant 9$, and (3) $m\geqslant 10$. For each of these classes we computed, for each of the four 45° intervals, the quantities P_1^* and P_2^*. From these estimates we computed for each 45° interval the quantities Y, defined by Eq. (13). They are given in Table VI, which shows that the deviations of Y from unity are negligibly small in all cases.

Table VI. Test for homogeneity with respect to business of the field of view: the quantity Y [Eq. (13)] estimated per quadrant for experiment H.

Quadrant	I	II	III	IV
Y	1.0016	0.9997	1.0007	1.0040

9. DISCUSSION

9.1 Our Results

It seems useful to discuss briefly the merits and shortcomings of each of our experiments.

Obviously, in spite of its very rich statistics, experiment L is most open to doubts, mainly because of its low scanning efficiencies. Therefore, most of our control experiments and tests have been performed on sample H.

In experiment H the scanning efficiencies have been substantially increased; the test for isotropy of reference α stars in the immediate vicinity of each π-μ-e event practically excludes effects of distortion on decay-angle distribution. Hence, we believe that the results of this experiment, supported by the negative result of all searches for imaginable inhomogeneities, prove that the anisotropy is genuine; as a fluctuation such a finding may be expected once in 2500 experiments.

This conclusion is strengthened by experiment T, which by virtue of its scanning method is free from any bias. Notwithstanding its relatively low statistics, it still yields a Pearson probability of only 0.7% for compatibility with isotropy, while its deviation from experiment H is measured by $\chi^2=5$ with 3 degrees of freedom[31] ($P_{\chi^2}=17\%$).

As to experiment K-5, it again shows anisotropy significantly by itself. The somewhat lower efficiencies as compared to experiment H are easily explained by the low density of events ($\sim$1 event in four fields of view).

A look at Fig. 5 shows, however, that a preferential loss of "straight" π-μ events is unlikely. This is confirmed by the double scan on the restricted area, which proves both that the detection efficiencies are practically independent of angle and that even with such low statistics (610 independent events) the bias-free Ω^* distribution is significantly nonisotropic ($P_{\chi^2}\approx 1\times 10^{-4}$).

We believe that—at least for the time being—the main significance of experiment K-5 lies in the fact that a departure from isotropy has been found also in another stack irradiated with another pion beam.

The agreement between the b values of the two experiments might well mean that the physical conditions were actually very similar in both exposures, but this may also be due to chance since so far we are unable to state which of the physical parameters are essential for the shape of the distribution.[32]

As to the d values, their equality is not to be expected even for physically identical distributions. It is easily understood that inclusion of steeper muon tracks decreases the value of $|d|$ for projected angles. This effect is evident in Table I (experiments T and K-5 contain dipping muons).

Future experiments, some of which are in preparation, will have to elucidate the shape of the distributions and their dependence on various physical parameters. If, e.g., the longitudinal polarization, implied by the forward-backward asymmetry observed in our stacks, could be shown to be present in the production reaction too,[33] this would imply nonconservation of parity in a strong interaction.

9.2 Results of Other Authors

A great deal of work has been done (over 7×10^4 detected π-μ decays) in a search for anisotropy of the muons. Unfortunately, as far as we are able to gather, in none of these experiments has a concentrated effort been made to look for *all* kinds of anisotropy (be it asymmetrical or not) and/or for *all* possible systematic errors.

Therefore, we felt it worthwhile to do so, within the restricted limits of our possibilities, especially since several authors, yielding to the general opinion, have formulated negative conclusions in spite of their own positive results.

[31] A. Hald, *Statistical Theory with Engineering Applications;* quoted from Russian translation, Moscow, 1956, p. 633.

[32] S. Ţiţeica, Rev. Phys. Acad. Rep. Populaire Roumaine 3, 171 (1958) has shown that for, e.g., spin 1 "pions" polarization is defined in the simplest case by sixteen parameters.

[33] This need not necessarily be true in view of filters and fields traversed from target to stack.

It is interesting to note that the most important paper of the Columbia group[7] contains two equally reliable, but conflicting results, obtained, respectively, in counter and emulsion experiments. As to the electronic π^+ experiment, their negative result is open only to the objection, mentioned already by the authors, that they could not detect even terms in $\cos\varphi$, which—as can be seen from our Table I—predominate mostly ($|d|>|b|$).

On the other hand, this same paper[7] contains a clearly positive result for the slow pions from τ decay in emulsion ($\chi^2\approx 9.3$ with one degree of freedom, i.e., $P_{\chi^2}\approx 2\times 10^{-3}$), under practically ideal experimental conditions, precluding any methodical objections.

The only other π-μ counter experiment[8] would have been able to detect only transversal polarizations. A comparison with Table I shows that the effects under consideration are too small to be detected at the quoted statistical and systematic inaccuracies.

It must be borne in mind that direct comparison of different results obtained under different experimental conditions is, generally speaking, inconclusive and of little use. Thus, for instance, a bending magnet might change the direction of polarization; a cyclotron field might depolarize those internally produced pions which initially were polarized; production in a many-body reaction may lead to polarization conditions essentially differing from those of a two-particle reaction, etc. (see, e.g., reference 14). The complex character of the phenomena is illustrated best by the energy dependence of the asymmetry in π-μ decay from τ mesons[7] and, perhaps, by the unusual behavior of pions from $K_{\pi 2}$ decay.[34] Hence, there is no reason to expect similar distributions, asymmetry coefficients, etc., in different experiments.

Before going into a detailed discussion of emulsion and chamber work, we want to stress that, in order to be conclusive, such experiments must include checks against—at least—three systematic errors, capable of simulating anisotropy but also of enhancing or diminishing an existing effect: (i) observational and geometrical bias; (ii) confusion of beam muons with stopped pions, or vice versa; (iii) distortion of the recording medium and/or of optics.

9.2(1) Bubble chamber experiments. In none of the papers[9–13] were such checks performed exhaustively. Even if the over-all efficiency has been estimated by double scan,[13] no estimation of efficiency per angular interval has been accomplished so that no bias correction could be made. No systematic measurements have been made in order to separate beam muons from straight π-μ decays. Comparison between the treatment of the 0°–20° angular interval in the experiments of references 11–13 shows convincingly that without such measurements the interpretation must remain largely arbitrary.

[34] G. Alexander, R. H. W. Johnston, and C. O'Ceallaigh, Nuovo Cimento **6**, 478 (1957).

It is difficult to understand how backward muons (180°) can escape observation in spite of the decay positron emerging from the μ track.[10–11] Here, again, only a double scan could justify such an assertion. Taking the data of, for instance, reference 10 at face value, a Pearson test against isotropy yields $\chi^2\approx 49$ with 9 degrees of freedom.

9.2(2) Emulsion experiments. A number of emulsion experiments are open to serious criticisms. For instance, in reference 15 the authors claim bias as an explanation of their significant lack of isotropy ($\chi^2=24$ for 3 degrees of freedom), based upon an arbitrary division of their data which has been shown in reference 6 to be statistically inadmissible.

The experiment of reference 16 has been quoted sometimes (see, e.g., reference 17) as an "experiment with good statistics," showing no forward-backward asymmetry. In fact, the uncorrected b value (-0.048 ± 0.020) of these authors[16] is not significantly at variance with our uncorrected result. Their corrected b value has been obtained by measuring the detection efficiencies on a very limited muon sample (~ 1500 events) and then applying the correction so obtained to the large sample of $\sim 10\,000$ events. Hence, the accuracy of the corrected result is essentially determined by the small sample of ~ 1500 events, and the quoted standard deviation of their corrected result ($b=0.009\pm 0.018$) is obviously erroneous. As can be shown, the actual standard deviation is at least ± 0.054. This means that the experiment of reference 16 neither proves nor disproves isotropy.

The experiment of reference 18 is inconclusive because: (i) The efficiencies have not been estimated for each of both scans separately and the bias elimination has not been carried through; (ii) muons have not been discriminated against; (iii) the stack used in this experiment has been shown in reference 19 to be strongly affected by distortion.

The experiment of reference 20 was performed on plates from our stack and would therefore seem to be most appropriate for a comparison. Unfortunately, the conclusions drawn in that paper are based on qualitative considerations. Here again, scanning efficiencies have not been estimated quantitatively, nor were beam muons investigated. None of the b values in reference 20 is at variance with our present results (experiments T and H). The comparison of our old results, uncorrected for bias,[4] with the results of reference 20, also uncorrected, seems of no practical interest.

Another group of authors[21,22,2] are unable to explain their clearly anisotropic results. The authors of reference 22 try to invoke too low statistics; in fact their data, if compared with isotropy, yield $\chi^2\approx 37$ with 17 degrees of freedom.

As to Lattes,[2] although all the critical checks against bias, beam muons, and distortion have shown that his projected angular distribution is free from such spurious effects, he states that "because of the peculiar dependence of the asymmetry coefficient on latitude, the departure from isotropy is not believed to be due to a real property of the pion." Or, using Fig. 11 of reference 2, it can be shown that: (i) 7 (or maybe 8) out of 20 values deviate from their weighted mean by more than one standard error, one of which values with more than two standard errors, as is expected in a normal error distribution; (ii) a Pearson test for consistency of these values with their weighted mean yields $\chi^2 \approx 16.5$ with 19 degrees of freedom, i.e., $P_{\chi^2} \approx 68\%$. Under such circumstances it seems hard to believe in a "peculiar dependence," and hence there are no objections left against the genuine character of the departure from isotropy in Lattes' projected angular distribution.

Finally, in reference 17 an experiment is described, in which—as far as we can see—all necessary precautions have been taken. Though from a purely statistical point of view the results are not at variance with ours, they are also well consistent with isotropy. Even if further enlarged statistics would prove isotropy, this would not invalidate our point of view. Indeed, there is no reason to expect the same angular distribution of the decay muons of pions from proton production at 680 MeV and from photoproduction at 1 GeV.

9.3 Concluding Remarks

After the critical analysis of the data from both our stacks (NIKFI and Ilford K-5), and the broad variety of checks performed, we are unable to ascribe the significant departure from isotropy to anything other than a genuine physical effect.

If account is taken also of the effectively positive results of references 2, 21, and 22, it seems hard to avoid the conclusion that at least in some of pion producing reactions part of these particles emerge with nonvanishing and orientated spin.

Further extensive experiments are needed to gain information on the role of the physical parameters possibly implied.

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APPENDIX I

Statistical Errors of Estimates Ω^*, P_1^*, and P_2^*

Consider a sample of emulsions, equal in volume, identical in geometrical respect, and irradiated under identical conditions.

Let each of these emulsions be expected to contain Ω events of a given specified kind. The probability that a given emulsion of the sample mentioned contains exactly Q events is given by the Poisson law:

$$W(Q|\Omega)=e^{-\Omega}\Omega^Q/Q!. \quad \text{(A1)}$$

In a double scan with efficiencies P_1 and P_2 of an emulsion which contains Q events, the probability of obtaining exactly D "double" events and, respectively, S_1 and S_2 "single" events is given by the polynomial law:

$$w_Q(S_1,S_2,D|Q,P_1,P_2)=\frac{Q!(P_1P_2)^D[P_1(1-P_2)]^{S_1}[P_2(1-P_1)]^{S_2}[1-P_1P_2-P_1(1-P_2)-P_2(1-P_1)]^{Q-D-S_1-S_2}}{D!S_1!S_2!(Q-D-S_1-S_2)!}. \quad \text{(A2)}$$

This distribution enables us to predict the probability with which a certain scanning result (i.e., the values S_1, S_2, and D) will be obtained in another double scan of the same emulsion, if the efficiencies P_1 and P_2 remain unchanged.

Practically, it is of more interest to know the probability with which the result (S_1, S_2, D) will be obtained in a double scan with efficiencies P_1 and P_2 of another emulsion which is one of the elements of the sample mentioned at the beginning of this Appendix. This probability is

$$w(S_1,S_2,D|\Omega,P_1,P_2)$$
$$=\sum_{Q=S_1+S_2+D}^{\infty} w_Q(S_1,S_2,D|Q,P_1,P_2)W(Q|\Omega)$$
$$=\frac{[\Omega P_1(1-P_2)]^{S_1}}{S_1!}\exp[-P_1(1-P_2)\Omega]$$
$$\times\frac{[\Omega P_2(1-P_1)]^{S_2}}{S_2!}\exp[-P_2(1-P_1)\Omega]$$
$$\times\frac{(P_1P_2\Omega)^D}{D!}\exp(-P_1P_2\Omega). \quad \text{(A3)}$$

In the usual way, from Eq. (A3) we obtain the likelihood function,

$$L(\Omega,P_1,P_2|S_1,S_2D)=\text{const}-\Omega(P_1+P_2-P_1P_2)$$
$$+N\ln\Omega+S_1\ln(1-P_2)+S_2\ln(1-P_1)$$
$$+(D+S_1)\ln P_1+(D+S_2)\ln P_2, \quad \text{(A4)}$$

where N is defined by Eq. (5) of Sec. 4.

The usual procedure yields the well-known results concerning the estimates for Ω, P_1, and P_2, viz., Eqs. (4) and (6) of Sec. 4.

The standard deviations of these estimates can be obtained[35] as the diagonal elements of the matrix $\|M^{-1}\|$, defined as

$$\|M\|\cdot\|M^{-1}\|=\|U\|, \quad \text{(A5)}$$

where U is the unit matrix and $\|M\|$ is the matrix of second partial derivatives of L with respect to Ω, P_1, P_2, taken at Ω^*, P_1^*, and P_2^*. Denoting derivatives with respect to Ω, P_1, and P_2 by lower indices 0, 1, 2, we have

$$\|M\|=\begin{Vmatrix}-L_{00} & -L_{10} & -L_{20}\\ -L_{01} & -L_{11} & -L_{21}\\ -L_{02} & -L_{12} & -L_{22}\end{Vmatrix}. \quad \text{(A6)}$$

The standard deviation of the estimates Ω^*, P_1^*, P_2^* are given by

$$\sigma^2(\Omega^*)=(L_{11}L_{22}-L_{12}^2)/\Delta, \quad \text{(A7)}$$
$$\sigma^2(P_1^*)=(L_{00}L_{22}-L_{02}^2)/\Delta, \quad \text{(A8)}$$

and an analogous expression for $\sigma^2(P_2^*)$, where

$$\Delta=-L_{00}L_{11}L_{12}-2L_{01}L_{02}L_{12}$$
$$+L_{00}L_{12}^2+L_{11}L_{02}^2+L_{22}L_{01}^2; \quad \text{(A9)}$$

all derivatives are to be taken at $\Omega=\Omega^*$, $P_{1,2}=P_{1,2}^*$.

After simple but lengthy calculations, Eqs. (A7) and (A8) can be brought to the explicit form:

$$\sigma^2(\Omega^*)=\Omega^*[1+(1-P_1^*)(1-P_2^*)/P_1^*P_2^*], \quad \text{(A7')}$$
$$\sigma^2(P_{1,2}^*)=P_{1,2}^*(1-P_{1,2}^*)/P_{2,1}^*\Omega^*. \quad \text{(A8')}$$

Equations (A7′) and (A8′) are identical with the corresponding expressions deduced in a recent paper[36] by other methods.

[35] R. Fisher, *Statistical Methods and Scientific Inference* (Oliver and Boyd, Edinburgh and London, 1956, p. 154).

[36] M. I. Podgoretzky and E. N. Tsyganov, J. I. N. R. Report P-839, Dubna 1961 (unpublished).